Using Intel Xeon Phi for Solving Non-Stationary Linear Programming Problems

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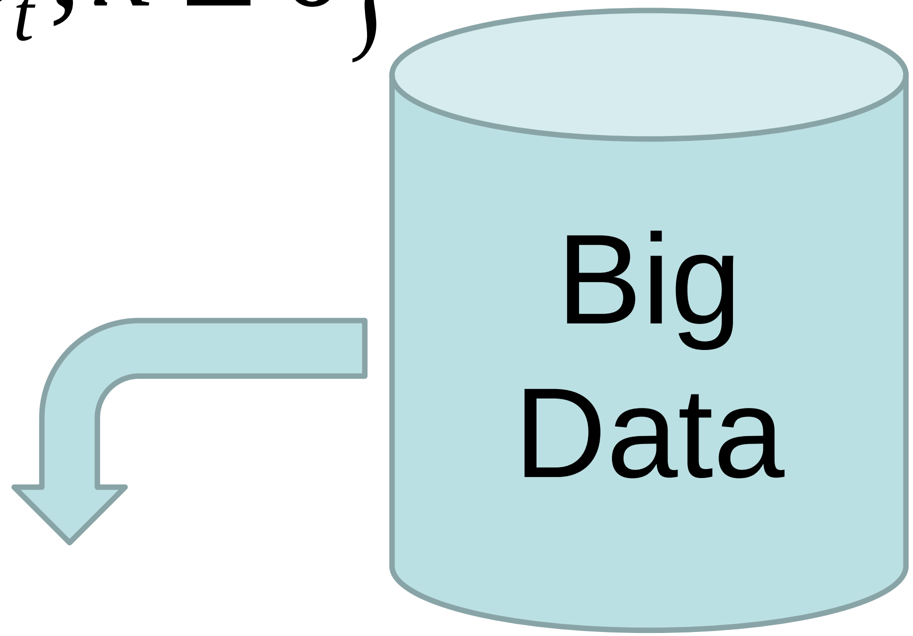
Non-stationary large-scale linear programming problem

$$\max\left\{\left\langle c_t, x\right\rangle \mid A_t x \le b_t, x \ge 0\right\}$$

- $x \in \mathbb{R}_n$
- A_t – matrix $m \times n$
- b_t – m-dimensional vectors
- c_t – n-dimensional vector
- $t \in \mathbb{R}_{\geq 0}$ – time



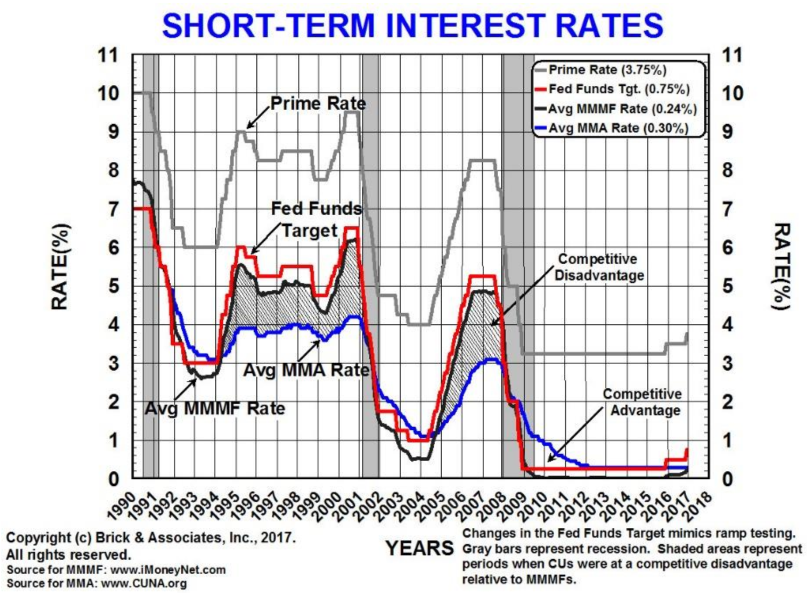
$n, m > 10^6$

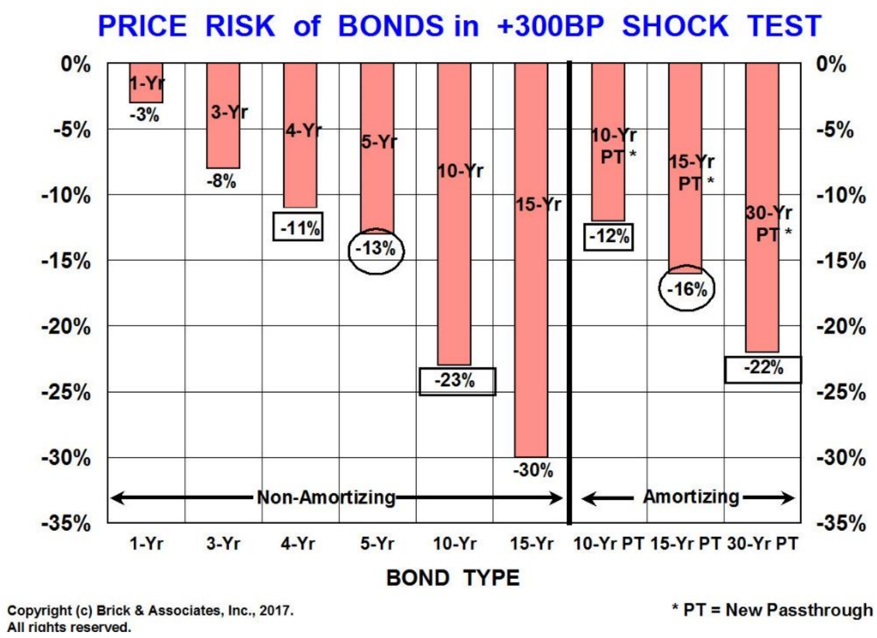
$Period\ of\ input\ data\ change < 10^{-2}$ sec.

Asset-Liability Management

Dynamic linear programming problem

- 1.7 billion inequalities
- 5.1 billion variables

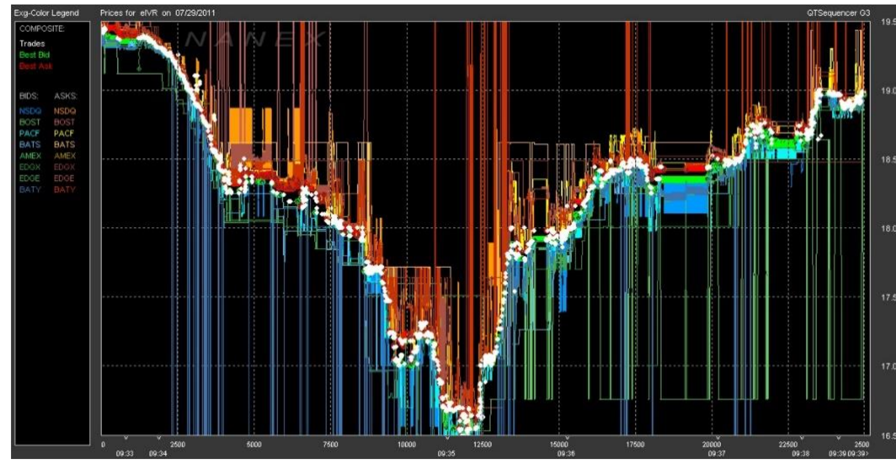


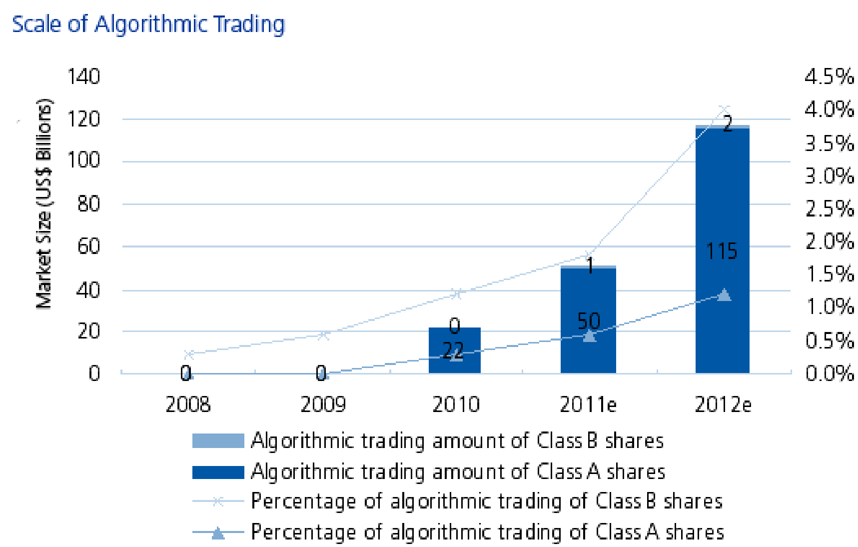


Sodhi M.S. LP modeling for asset-liability management: A survey of choices and simplifications // Operations Research. 2005. V. 53. No. 2. P. 181-196.

High Frequency Trading

- Dimension: 10^5-10^6
- Number of inequalities: 10^6-10^7
- Period of input data change: 10^{-2} - 10^{-3} sec.

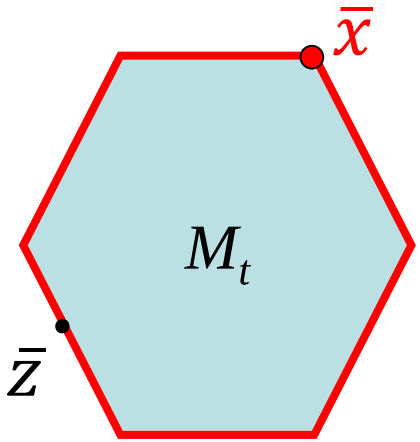




NSLP Algorithm (Non Stationary Linear Programming)

Algorithm phases:

- *Quest* – find point $\bar{z} \in M_t$
- *Targeting* – moving point $\bar{z}$ in such a way that the solution $\bar{x}$ of LP problem remains permanently in an ε-vicinity of $\bar{z}$



$$A_t x \le b_t \iff x \in M_t$$

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(Non Stationary Linear Programming)

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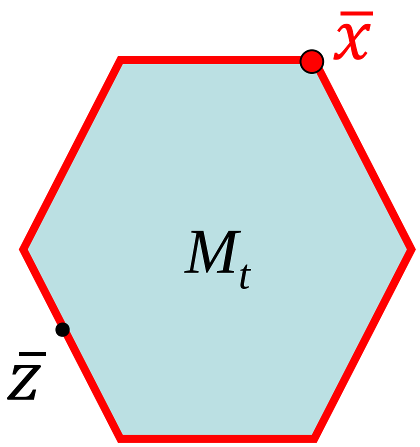
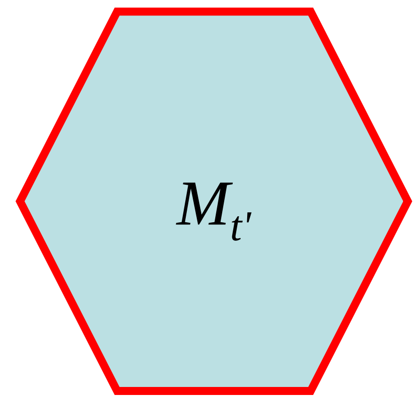


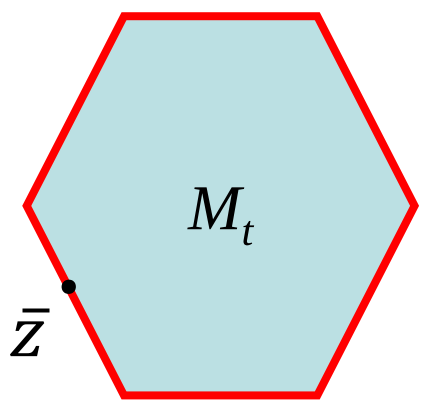
$$A_t x \le b_t \;\Leftrightarrow\; x \in M_t$$

Quest Phase (Finding $\bar{z} \in M_t$)

Conventional methods for solving a system of linear equalities can not give a solution to the problem $A_t x = b_t$ because of its non-stationarity.



$$A_{t'} x \leq b_{t'} \Leftrightarrow x \in M_{t'}$$



$$A_t x \leq b_t \Leftrightarrow x \in M_t$$

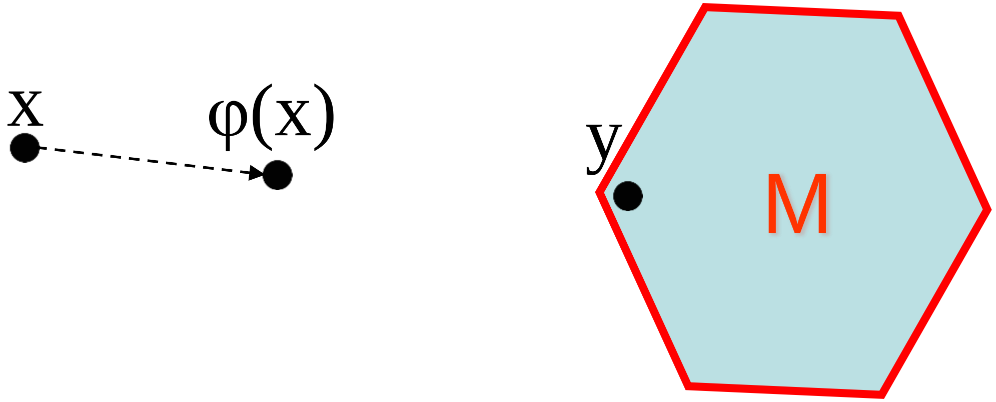
Fejerian Map

M – convex bounded set

$\varphi \in \{R^n \to R^n\}$ – M-fejerian map if

$$\varphi(y) = y, \ \forall y \in M ;$$

$$\|\varphi(x) - y\| < \|x - y\|, \ \forall y \in M, \ \forall x \notin M .$$



Lipót Fejér
1880 – 1959

Fejerian Map for *Quest* Phase

$$\varphi_t(x) = x - \frac{1}{m}\sum_{i=1}^{m} \frac{\max\{\langle a_{ti}, x\rangle - b_{ti}, 0\}}{\|a_{ti}\|^2} \cdot a_{ti}$$

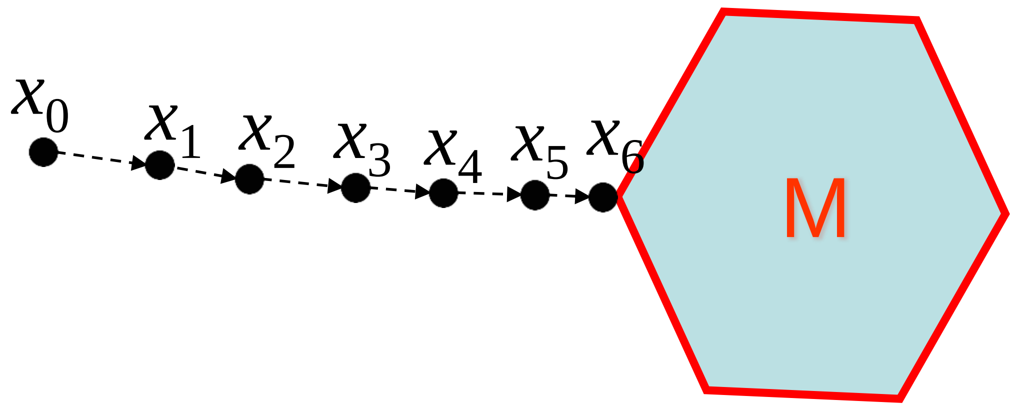
- a_{ti} – i-th line of matrix A_t
- $b_{t1}, \dots, b_{tm}$ – elements of column b_t
- m – number of lines in A_t
- t – time

Fejerian Process

$$\varphi^{s}(x) = \underbrace{\varphi\ldots\varphi(x)}_{s}$$

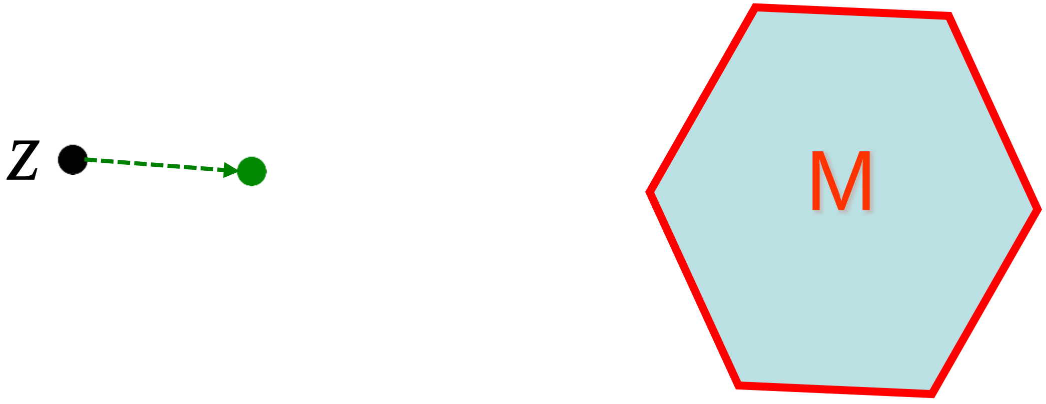
$$x_0 \in \mathbb{R}^n$$

$$\{\varphi^{s}(x_0)\}_{s=0}^{+\infty}$$



$$x_i = \varphi^{i}(x_0)$$

«Self-guidance» of Fejerian Process



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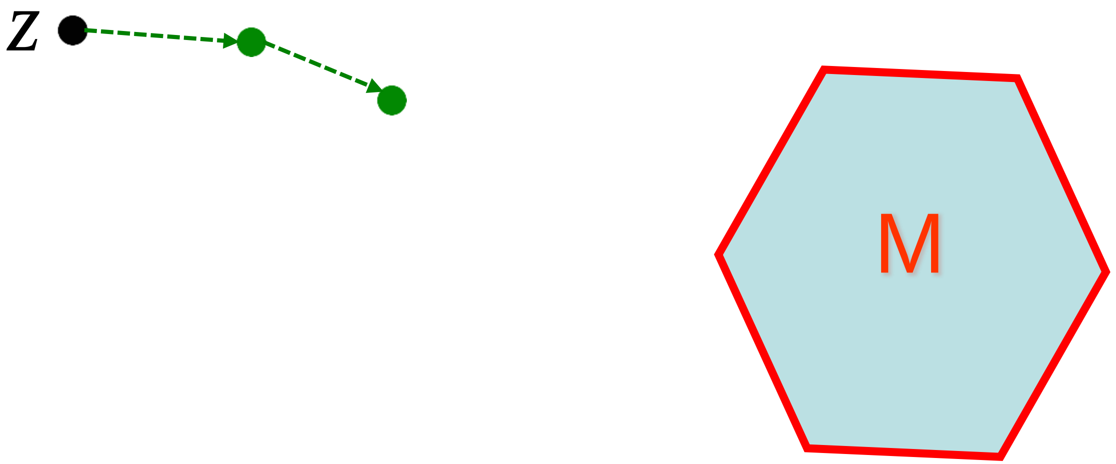

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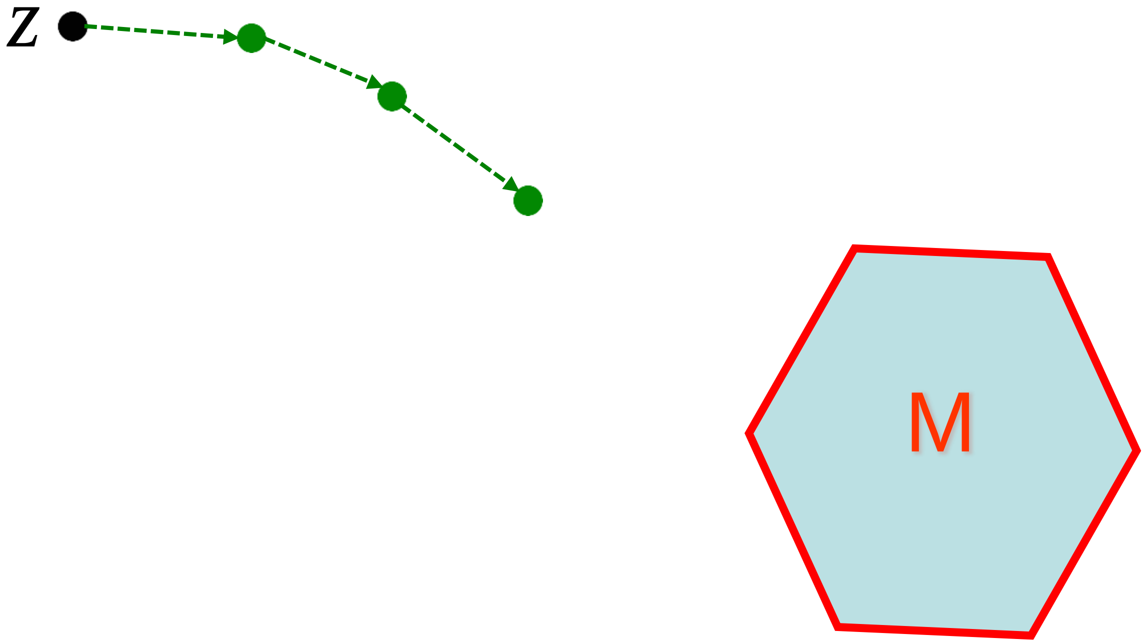

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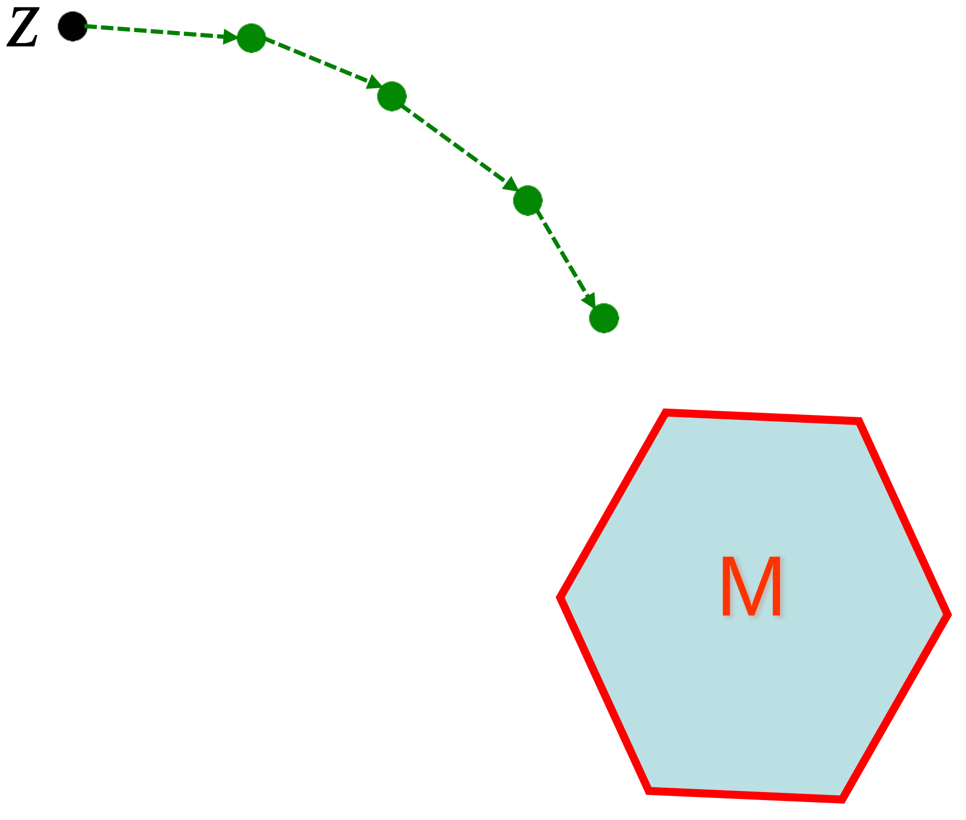

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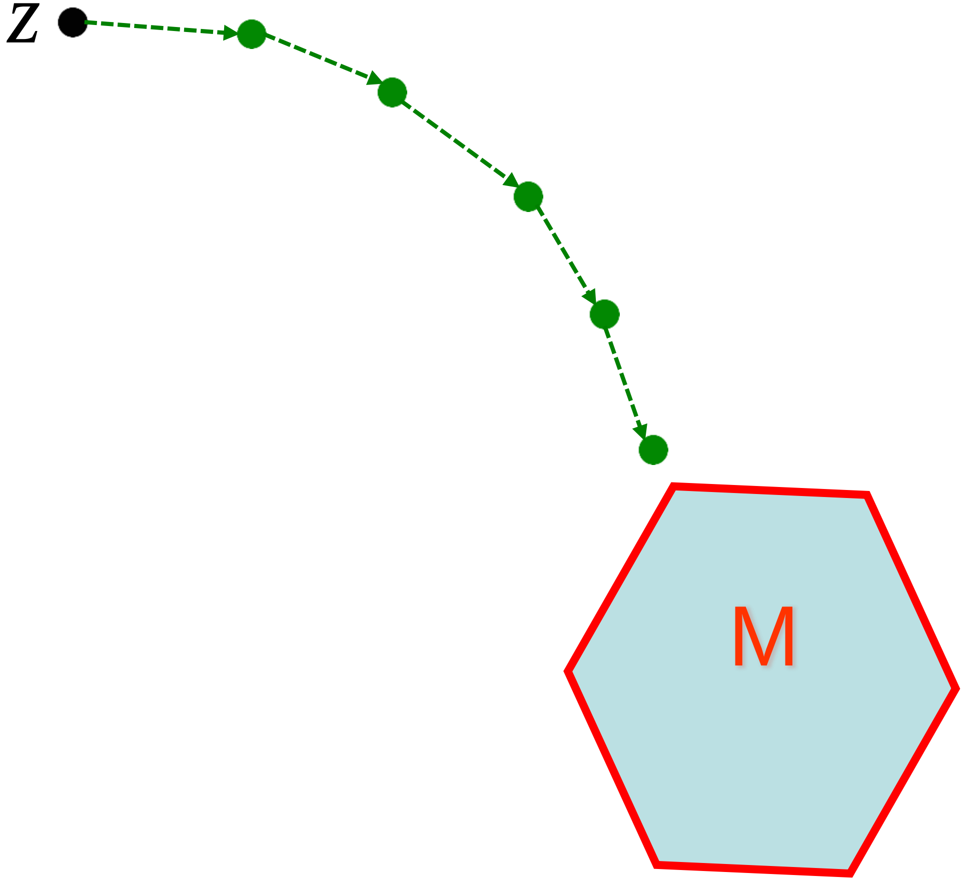

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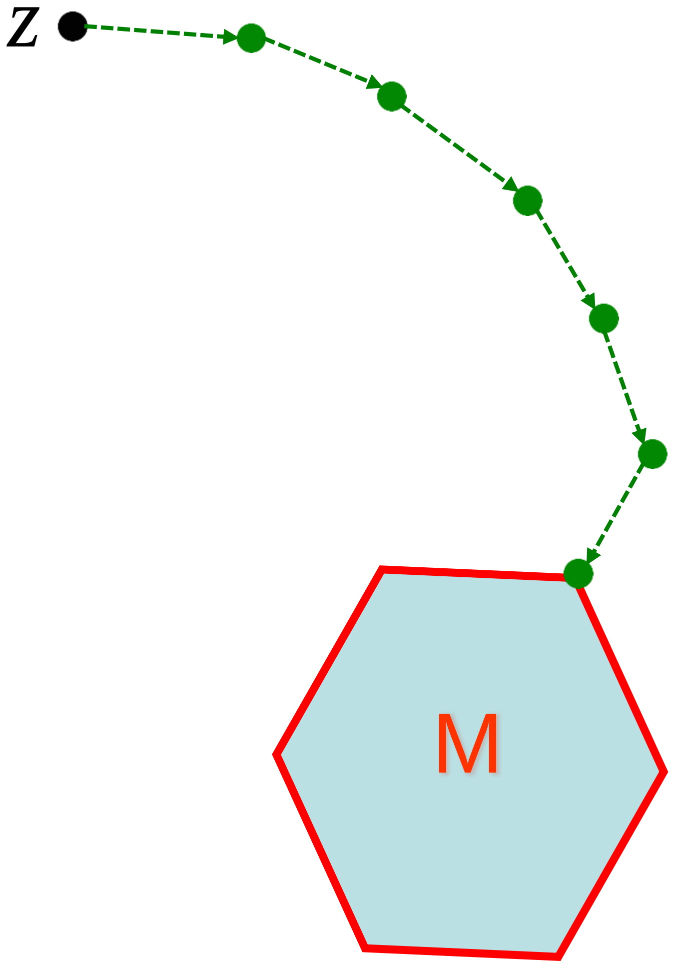


Scalable Synthetic LP Problem

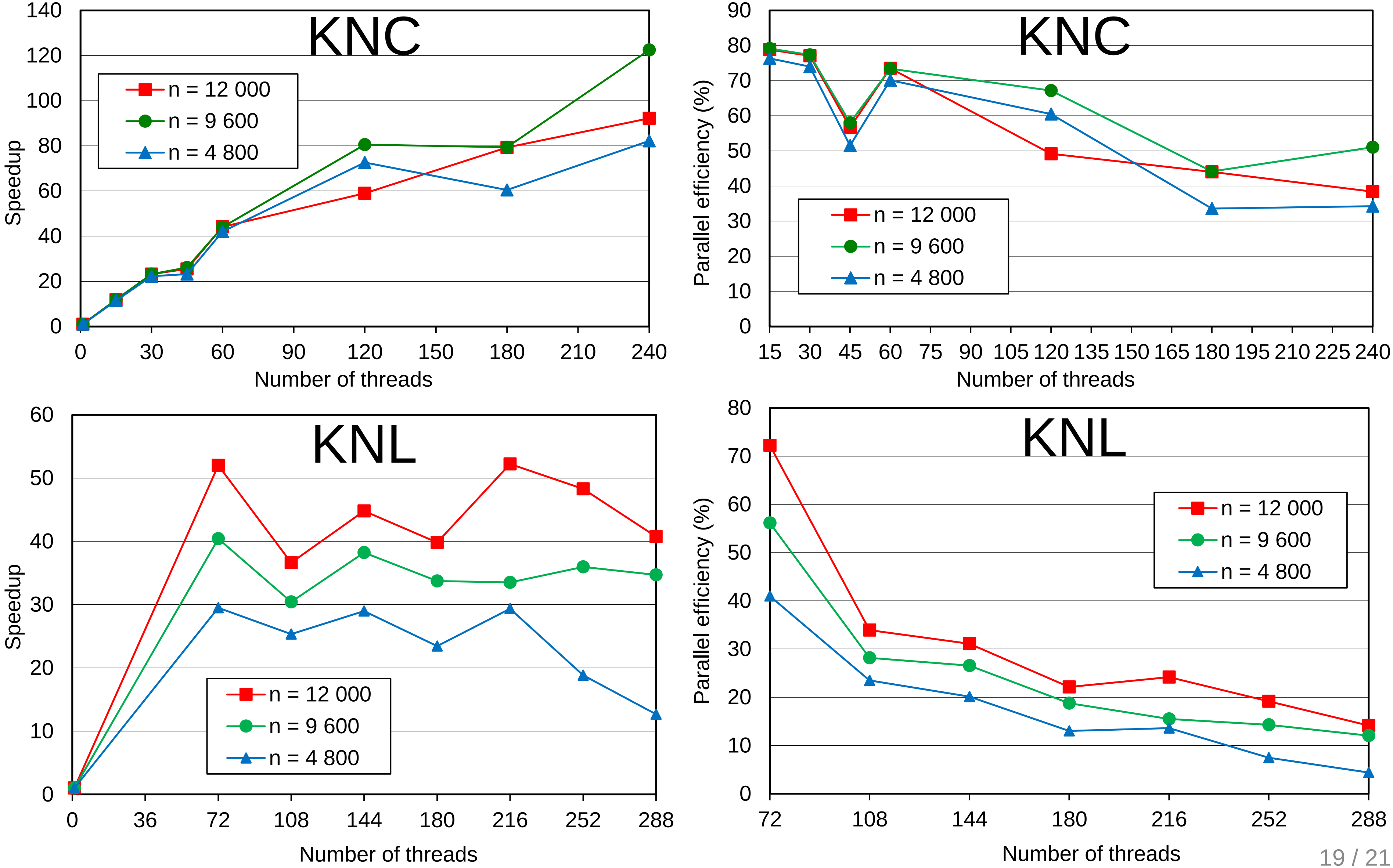
$$
\begin{cases}
x_0 & & & & & \leq & 200 \\
& & x_1 & & & \leq & 200 \\
& & & \ddots & & \cdots & \cdots \\
& & & & x_{n-1} & \leq & 200 \\
x_0 & + & x_1 & \cdots & + \; x_{n-1} & \leq & 200(n-1)+100 \\
x_0 & + & x_1 & \cdots & + \; x_{n-1} & \geq & 100 \\
x_0 & & & & & \geq & 0 \\
& & x_1 & & & \geq & 0 \\
& & & \ddots & & \cdots & \cdots \\
& & & & x_{n-1} & \geq & 0
\end{cases}
$$

$$
Q_{\max}(x) = 2x_0 + 2x_1 + \ldots + 2x_{n-2} + x_{n-1}
$$

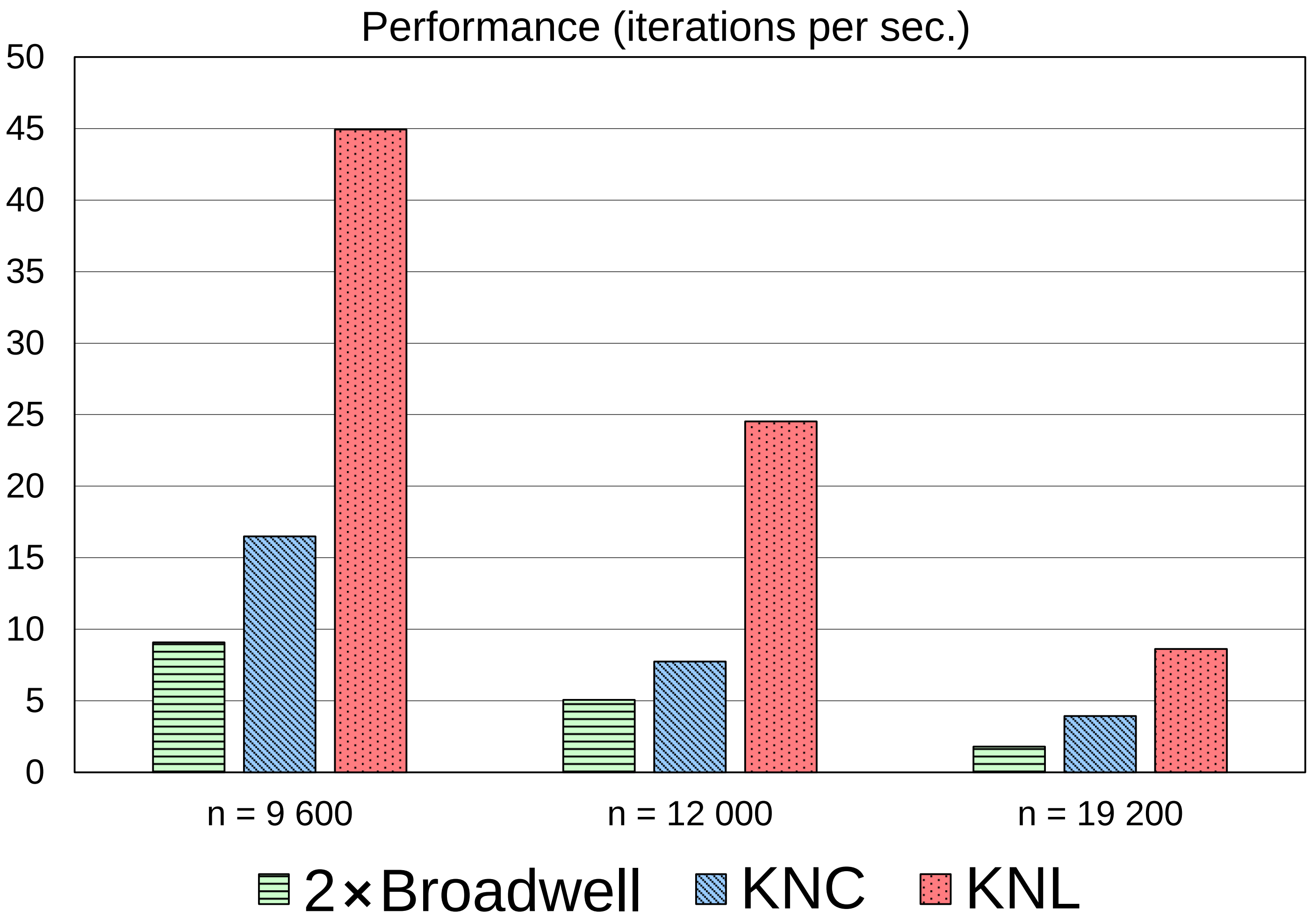
Benchmarked Processors

Code Name	Specifications
KNC	Intel Xeon Phi SE10X (61 cores, 1.1 GHz; 4 threads per core)
KNL	Intel Xeon Phi 7290 (72 cores, 1.5 GHz; 4 threads per core)
2×Broadwell	2×Intel Xeon E5 2697v3 32 cores, 2.6 GHz; 2 threads per core)

Speedup & Parallel Efficiency



Benchmark


Performance (iterations per sec.)
50
45
40
35
30
25
20
15
10
5
0
n = 9 600
n = 12 000
n = 19 200
2×Broadwell
KNC
KNL

Questions?